Solutions to the Cauchy-Poisson problem in 2D and 3D

Per A. Madsen

Department of Mechanical Engineering, Technical University of Denmark, Kgs Lyngby, 2800, Denmark.

email: prm@mek.dtu.dk

Abstract:

Tsunamis propagating in the deep ocean are basically linear weakly-dispersive transient waves, which develop their characteristic features over long distances. This motivates a study of the classical Cauchy-Poisson problem concerning development of linear waves generated by an impulsive initial condition.

The original work goes back to Poisson (1816), but more recent formulations have been presented by e.g. Wehausen & Laitone (1960), Whitham (1974), LeBlond & Mysak (1978) and Clarisse et al. (1995). The resulting solutions for the surface elevation can be expressed as a Fourier integral in two dimensions and a Fourier-Bessel integral in three dimensions. For large values of time and large distances from the initial disturbance, these integrals can be approximated by asymptotic expansions, which eventually lead to a formulation in terms of Airy functions and their derivatives.

In this work, we consider two different asymptotic approximations to the exact integral formulation for a delta-function initial disturbance. Method 1 is based on Whitham's local solution valid near the front of the wave train, which is equivalent to a KdV-type approximation to the linear dispersion relation. Method 2 is based on a uniform transformation of the variables of integration combined with the method of stationary phase for oscillating integrals and a determination of the resulting Bleistein sequences. Both methods are developed in 2D and 3D, and the corresponding impulse response functions are presented. The two approximative methods are compared to direct numerical integration of the exact formulations.

Finally, we present the convolution formulations in 2D and 3D, which cover the case of general initial conditions. We study the cases of an initial Gaussian disturbance (narrow as well as wide) and an initial rectangular disturbance. The results are compared to numerical simulations using a high-order Boussinesq model.

References:

1. Clarisse, J.-M., Newman, J.N. and Ursell, F., Proc. Roy. Soc.Lond. A, 450, 67-87 (1994).
2. LeBlond, P.H. and Mysak, L.A., Waves in the ocean, Elsevier, New York (1978).
3. Poisson, S.D., Mmoire sur la thorie des ondes. Mm. Acad. R. Sci., 1, 71-186 (1816).
4. Wehausen, J.V. and Laitone, E.V., In Encyclopedia of physics, 9, 446-778 (1960).
5. Whitham, G.B., Linear and nonlinear waves, New York: Wiley-Interscience (1974).